**CBCS/ SEMESTER SYSTEM**

**(w.e.f. 2020-21 Admitted Batch)**

**B.A./B.Sc. MATHEMATICS**

**COURSE-III, ABSTRACT ALGEBRA**

**Time: 3Hrs Max.Marks:75M**

**SECTION - A**

**Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M**

1. Show that the set G= is a group under multiplication

2. Define order of an element. In a group G, prove that if then .

3. If H and K are two subgroups of a group G, then prove that HK is a subgroup 

HK=KH

4. If G is a group and H is a subgroup of index 2 in G then prove that H is a normal subgroup.

5. Examine whether the following permutations are even or odd i)

ii)

6. Prove thata group of prime order is cyclic.

7.Prove that the characteristic of an integral domain is either prime or zero.

8. If F is a field then prove that and F are the only ideals of F.

**SECTION - B**

**Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M**

9 a) Show that the set of nth roots of unity forms an abelian group under multiplication.

(Or)

9 b) In a group G, for , O(a)=5, b ≠ e and . Find O(b).

10 a) The Union of two subgroups is also a subgroup one is contained in the other.

(Or)

b) State and prove Langrage’s theorem.

11 a) Prove that a subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

(Or)

11 b) State and prove fundamental theorem of homomorphisms of groups.

12 a) Let Sn be the symmetric group on n symbols and let An be the group of even permutations. Then show that An is normal in Sn and O(An ) =

(Or)

12 b)prove thatevery subgroup of cyclic group is cyclic.

13 a) Prove that every finite integral domain is a field.

(Or)

13 b) Define principal idea. Prove that every ideal of Z is a principal ideal.